

Touch count and remove the number to be taken away, in this case 4.


If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown. Understand subtraction as take-away:


Understand subtraction as finding the difference:


The above bar model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation. The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

Year 2
Missing number problems e.g. 52-8= $\quad$; $-20=$
25; $22=-21 ; 6+\square+3=11$
It is valuable to use a range of representations (also see Y1). Continue to use number lines to model takeaway and difference. E.g.


The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25 .


The bar model should continue to be used, as well as images in the context of measures.

## Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. 75 42


Year 3
Missing number problems e.g. $\quad=$ 43-27; 145- $=$ 138; 274-30 = $: 245-\square=195 ; 532-200=\square$; 364$153=$
Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving (see Y 1 and Y 2 ).
Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

## Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation)


For some children this will lead to exchanging, modelled using place value counters (or Dienes).
A number line and expanded column method may be compared next to each other.


Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

# EXTRA GUIDANCE - PROGRESSION IN REASONING -SUBTRACTION 

| Year 1 |
| :--- |
| Mental Strategies <br> Children should experience regular counting on <br> and back from different numbers in 1s and in |

## Mental Strategies

Children should count regularly, on and back, in steps of 2,3,5 and 10. Counting back in tens from any number should lead to subtracting multiples of 10 . Number lines should continue to be an important image to support thinking, for example to model how to subtract 9 by adjusting.


Children should practise subtraction to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g using 10-7=3 and $7=10-3$ to calculate $100-70=30$ and $70=$
100-30.


As well as number lines, 100 squares could be used to model calculations such as 74-11,77-9 or 3614 , where partitioning or adjusting are used. On the example above, 1 is in the bottom left corner so that 'up' equates to 'add'.
Children should learn to check their calculations, including by adding to check.

## Mental Strategies

Children should continue to count regularly, on and back, now including multiples of $4,8,50$, and 100 , and steps of 1/10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.
Children should continue to partition numbers in difference ways.
They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for 201-198; counting back (taking away / partition into tens and ones) for 201-12.
Calculators can usefully be introduced to encourage fluency by using them for games such as 'Zap' [e.g. Enter the number 567. Can you 'zap' the 6 digit and make the display say 507 by subtracting 1 number?]
The strategy of adjusting can be taken further, e.g. subtract 100 and add one back on to subtract 99 .
Subtract other near multiples of 10 using this strategy.

## Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y 1 and y 2

## Generalisations

Noticing what happens to the digits when you count in tens and hundreds.
Odd - odd = even etc (see Year 2)
Inverses and related facts - develop fluency in finding related addition and subtraction facts.

## Subtraction, subtract, take away, distance

 between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,
## Generalisations

True or false? Subtraction makes numbers smaller
When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions. Children could see the image below and consider, "What can you see here?" e.g.
3 yellow, 1 red, 1 blue.
$3+1+1=5$
2 circles, 2 triangles, 1
square. $2+2+1=5$
I see 2 shapes with

curved lines and 3 with
straight lines. $5=2+3$
$5=3+1+1=2+2+1=2+3$

## Some Key Questions

How many more to make...? How many more is... than...? How much more is...? How many are left/left over? How many have gone? One less, two less, ten less... How many fewer is... than...? How much less is...?
What can you see here?
Is this true or false?

## Problem Solving using the Singapore Bar

## Method

Using images to begin with, moving to a more abstract representation when secure of the comparison model.E.g. Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have?


They should continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up.
They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. $23=20+3=10+13$.

## Vocabulary

Subtraction, subtract, take away, difference, difference between, minus
Tens, ones, partition
Near multiple of 10, tens boundary
Less than, one less, two less... ten less... one hundred less.... More, one more, two more... ten more... one

## hundred more

## Generalisation

Noticing what happens when you count in tens (the digits in the ones column stay the same)
Odd - odd = even; odd - even = odd; etc show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.


Some Key

## Questions

## How many more to make...? How many more is...

 than...? How much more is...? How many are left/left over? How many fewer is... than...? How much less is...?Is this true or false?
If I know that $7+2=9$, what else do I know? (e.g. 2 $+7=9 ; 9-7=2 ; 9-2=7 ; 90-20=70 \mathrm{etc})$. What do you notice? What patterns can you see?

Develop the knowledge that the inverse relationship can be used as a checking method.

## Key Questions

What do you notice? What patterns can you see?
When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line

| $448-223$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 225 | 228 | 24,8 | 448 | $\begin{aligned} & 400 \\ & 200 \end{aligned}$ | $\begin{aligned} & 40 \\ & 20 \end{aligned}$ | $\begin{aligned} & 8 \\ & 3 \\ & \hline \end{aligned}$ |
|  |  |  |  |  |  |  |
|  |  |  |  | 200 | 20 | 5 |
|  |  |  |  |  | $=2$ |  |




EXTRA GUIDANCE - PROGRESSION IN REASONING -SUBTRACTION

| Year 4 |
| :--- |
| $\frac{\text { Mental Strategies }}{\text { Children should continue to count regularly, on and back, }}$, |

Children should continue to count regularly, on and back, now including multiples of $6,7,9,25$ and 1000, and steps of $1 / 100$.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.
Children should continue to partition numbers in different ways. They should be encouraged to choose from a range of strategies:
Counting forwards and backwards: 124-47, count back 40 from 124, then 4 to 80 , then 3 to 77
Reordering: $28+75,75+28$ (thinking of 28 as $25+3$ )
Partitioning: counting on or back: $5.6+3.7,5.6+3+0.7$
$=8.6+0.7$
Partitioning: bridging through multiples of 10: 6070$4987,4987+13+1000+70$
Partitioning: compensating - $138+69,138+70-1$
Partitioning: using 'near' doubles $-160+170$ is double 150 , then add 10 , then add 20 , or double 160 and add 10 , or double 170 and subtract 10
Partitioning: bridging through 60 to calculate a time interval - What was the time 33 minutes before 2.15 pm ? Using known facts and place value to find related facts.

## Mental Strategies

Children should continue to count regularly, on and back, now including steps of powers of 10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.
Children should continue to partition numbers in different ways.
They should be encouraged to choose from a range of strategies:
Counting forwards and backwards in tenths and hundredths: $1.7+0.55$
Reordering: $4.7+5.6-0.7,4.7-0.7+5.6=4+5.6$
Partitioning: counting on or back $-540+280,540+200$
$+80$
Partitioning: bridging through multiples of 10:
Partitioning: compensating: $5.7+3.9,5.7+4.0-0.1$
Partitioning: using 'near' double: $2.5+2.6$ is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
Partitioning: bridging through 60 to calculate a time interval: It is 11.45 . How many hours and minutes is it to 15.20?

Using known facts and place value to find related facts. Vocabulary

Year 6

## Mental Strategies

Consolidate previous years.
Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. $20-5 \times 3=5 ;(20-5) \times 3=45$

## Vocabulary

See previous years

## Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller.

## Some Key Questions

What do you notice?
What's the same? What's different?
Can you convince me?

## Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

## Generalisations

Investigate when re-ordering works as a strategy for subtraction. Eg. 20-3-10=20-10-3, but 3-20-10 would give a different answer.

## Some Key Questions

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?
tens of thousands boundary,
Also see previous years

## Generalisation

Sometimes, always or never true? The difference
between a number and its reverse will be a multiple of 9 .
What do you notice about the differences between consecutive square numbers?
Investigate $a-b=(a-1)-(b-1)$ represented visually.

## Some Key Questions

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

How do you know?



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